The Liquidity Coverage Ratio and Monetary Policy Implementation

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Abstract

In addition to revamping existing rules for bank capital, Basel III introduces a new global framework for liquidity regulation. One part of this framework is the Liquidity Coverage Ratio (LCR), which requires banks to hold sufficient high-quality liquid assets to survive a 30-day period of market stress. As monetary policy typically involves targeting the interest rate on loans of one of these assets – central bank reserves – it is important to understand how this regulation may impact the efficacy of central banks' current operational frameworks. We extend a standard model of monetary policy implementation in a corridor system to include term funding and an LCR requirement. When banks face the possibility of an LCR shortfall, we show that it becomes more challenging for a central bank to control the overnight interest rate and that the very short end of the yield curve becomes steeper. These results suggest that central banks may want to adjust their operational frameworks as the new regulation is implemented.

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"RBS's liquidity position at end-August 2008 would have translated to an LCR ... of between 18% and 32%, ... RBS would, therefore, have had to increase by between £125bn and £166bn its stock of high-quality unencumbered liquid assets or..."

The failure of the Royal Bank of Scotland - Financial Services Authority Board Report

1 Introduction

In response to the recent global financial crisis, the Basel Committee on Banking Supervision (BCBS) published a new international regulatory framework for banks, known as Basel III, in December 2010. In addition to revamping the existing capital rules, Basel III introduces – for the first time – a global framework for liquidity regulation. The new regulation prescribes two separate, but complementary, minimum standards for managing liquidity risk: the *Liquidity Coverage Ratio* (LCR) and the *Net Stable Funding Ratio* (NSFR). These standards aim to ensure that banks hold a more liquid portfolio of assets and reduce the maturity mismatch between their assets and liabilities. Specifically, the LCR requires each bank to hold a sufficient quantity of highly-liquid assets to survive a 30-day period of market stress. The NSFR focuses on a one-year time horizon and establishes a minimum amount of stable funding each bank must obtain based on the liquidity characteristics of its assets and activities. The LCR and NSFR are scheduled to be implemented in January 2015 and January 2018, respectively.

How might these new liquidity regulations affect the process through which central banks implement monetary policy? In many jurisdictions, this process involves setting a target for the interest rate at which banks lend central bank reserves to one another, typically overnight and on an unsecured basis. Because these reserves are part of banks' portfolio of highly-liquid assets, the regulations will potentially alter behavior in the interbank market, changing the relationship between market conditions and the resulting interest rate. In addition, monetary policy operations will modify banks' regulatory liquidity ratios and, hence, may influence their compliance with the new liquidity standards, at least at the margin. These linkages suggest that the new regulations may have subtle and intended consequences that could potentially reduce the effectiveness of central banks' current operating procedures.

We extend a standard model of banks' reserve management to study how the introduction of

an LCR requirement affects the process of implementing monetary policy in a corridor system. We show that when banks face the possibility of an LCR shortfall in some states of nature, the relationship between the quantity of central bank reserves and market interest rates changes dramatically. A bank that is concerned about possibly violating the LCR has a stronger incentive to seek term funding in the market and is more likely to borrow from the central bank. Both of these actions add to the bank's reserve holdings and thus lower the need to seek funds in the overnight market to ensure the bank's reserve requirement is met. This lower demand for overnight funds tends to drive down the overnight rate, whereas the increased demand for term funding tends to make the very short end of the yield curve steeper. In some cases, the overnight rate lies on the floor of the corridor *regardless* of the supply of reserves, indicating that a central bank's ability to control this interest rate using open market operations may be severely compromised.

We also study how open market operations alter the aggregate balance sheet of the banking system and thereby affect the likelihood that individual banks may face an LCR deficiency. These operations can be structured in a variety of different ways: as outright purchases (or sales) of assets or as reverse operations, with differing categories of assets eligible for purchase or for use as collateral, and with different types of counterparties. The structure of an operation determines its effects on bank balance sheets and, hence, on banks' incentives to trade in interbank markets. As a result, in a regime with an LCR, it is not only the size of an open market operation that matters for determining interest rates, but also the manner in which the operation is conducted. We show how different types of operations affect both the overnight and term interest rate in our model.

Our results indicate that central banks may need to adjust their operational frameworks for implementing monetary policy when the LCR is introduced. The size of the potential problem will depend on how closely banks manage their LCR to the minimum requirement, an issue that is outside the scope of our model. Nevertheless, our results indicate that the effects may be substantial, at least in some situations. The remainder of the paper is structured as follows. In the next section, we briefly review the LCR regulation. We then we present our model, which extends the standard model of monetary policy implementation in a corridor system to include term interbank lending and an LCR requirement. In Section 4, we derive the equilibrium interest rates in the overnight and term interbank markets and illustrate how the LCR changes the relationship between these rates and the supply of reserves. In Section 5, we introduce open market operations and study the degree to which the central bank can control interbank interest rates in the presence of an LCR. Section 6 concludes with a discussion of policy options.

2 The Liquidity Coverage Ratio

The Liquidity Coverage Ratio (LCR) builds on traditional methodologies used internally by banks to assess exposure to contingent liquidity events. The regulation requires that a bank's stock of *unencumbered* high-quality liquid assets (HQLA) be larger than projected net cash outflows over a 30-day horizon under a stress scenario specified by supervisors

$$LCR = \frac{Stock \ of \ unencumbered \ high-quality \ liquid \ assets}{Total \ net \ cash \ outflows \ over \ the \ next \ 30 \ calendar \ days} \ge 100\%.$$
(1)

Two types (or "levels") of assets can be applied toward the HQLA pool. Level 1 assets include cash, central bank reserves and debt securities issued or guaranteed by sovereigns, central banks or public sector enterprises with a 0% capital risk weight under Basel III.¹ Level 2 assets include debt securities issued by public authorities with a 20% risk weight under Basel III, plus highly rated non-financial corporate bonds and covered bonds. Level 2 assets can only comprise up to 40% of a bank's total stock of HQLA.

The denominator of the LCR, total expected net cash outflows, is calculated by multiplying the size of various types of liabilities and off-balance sheet commitments by the rates at which they are expected to run off or be drawn down in the stress scenario. This scenario includes a partial loss of retail deposits, significant loss of unsecured and secured wholesale funding, contractual outflows from derivative positions associated with a three-notch ratings downgrade, and substantial calls on off-balance sheet exposures. The calibration of scenario run-off rates reflects a combination of

¹Central bank reserves held to meet reserve requirements may be included in HQLA, to the extent that these reserves can be drawn down in times of stress. The LCR rules text states that "[1]ocal supervisors should discuss and agree with the relevant central bank the extent to which central bank reserves should count towards the stock of liquid assets, i.e., the extent to which reserves are able to be drawn down in times of stress".

the experience during the recent financial crisis, internal stress scenarios of banks, and existing regulatory and supervisory standards. From these outflows, banks are permitted to subtract expected inflows for 30 calendar days into the future. In order to prevent banks from relying solely on anticipated inflows to meet their liquidity requirement, and to ensure a minimum level of liquid asset holdings, the fraction of outflows that can be offset this way is capped at 75%.

While the mechanics of calculating the LCR are somewhat intricate, the objective is simple. As described by the Group of Central Bank Governors and Heads of Supervision (GHOS, which oversees the work of the Basel Committee on Banking Supervision) in a January 2012 press release,

"(t)he aim of the Liquidity Coverage Ratio is to ensure that banks, in normal times, have a sound funding structure and hold sufficient liquid assets such that central banks are asked to perform only as lenders of last resort and not as lenders of first resort."

Our goal in this paper is to shed light on how the introduction of an LCR may inadvertently affect the process by which central banks implement monetary policy.

3 The Basic Framework

Our analysis is based on a model of reserve management in the tradition of Poole (1968).² The basic setup loosely follows Bartolini, Bertola and Prati (2002), to which we add both an interbank market for term loans and liquidity regulation in the form of a Liquidity Coverage Ratio (LCR) requirement. Banks raise capital and issue deposits while holding loans, bonds and reserves as assets. They can borrow and lend funds in interbank markets for overnight and term loans. In addition to the LCR, banks are also subject to a reserve requirement. The central bank conducts monetary policy by setting a target for the interest rate in the overnight interbank market and, hence, a key element of the model is banks' demand for funds in this market. The central bank influences activity in interbank markets using a combination of open market operations and standing facilities where banks can deposit surplus funds or borrow against collateral.

²Contributions to this literature include Dotsey (1991), Guthrie and Wright (2000), Bartolini, Bertola and Prati (2002), Clouse and Dow (2002), Bindseil (2004), Whitesell (2006), Ennis and Weinberg (2007), Ennis and Keister (2008), Bech and Klee (2009), and Afonso and Lagos (2012), among others.

3.1 Balance Sheets and Payment Shocks

There are many identical banks, each of which is a price-taker in the interbank markets and aims to maximize expected profits. It is convenient to consider the decision problem facing a representative bank, which begins the day with a balance sheet of the following form:

Asset	s	Liabilitie	\mathbf{s}
Loans	L	Deposits	D
Bonds	В		
Reserves	R	Equity	E

The values of these variables are determined by activities that are outside of the scope of the model. One can, for example, think of these values as resulting from the bank's activity on behalf of customers. Markets for interbank lending then open. Banks can trade two types of contracts: overnight and term. Term contracts have a duration longer than 30 days, which implies they are treated differently from overnight loans for LCR purposes. Let Δ and Δ_T denote the amounts the bank borrows in the overnight and term market, respectively; negative values of these variables correspond to lending. The bank's balance sheet after the interbank market closes is

A	Assets	Liabilities		
Loans	L	Deposits	D	(3)
Bonds	В	Net interbank borrowing	$\Delta + \Delta_T$	(0)
Reserves	$R + \Delta + \Delta_T$	Equity	E	

The liability side now has a new category, "net interbank borrowing," which is the counterpart to the inflow (or outflow) of reserves from trading in the interbank market. Note that net interbank borrowing can be either positive or negative, and hence $(\Delta + \Delta_T)$ can either be a liability or an asset (i.e., a claim on other banks).

After the interbank market has closed, the bank experiences a payment shock in which an amount ε of customer deposits is sent as a payment to another bank. If ε is negative, the shock

represents an unexpected inflow of funds. The value of ε is independent across banks and is drawn from a symmetric distribution G with a mean of zero. The assumption that the interbank market closes before these payment shocks are realized is a standard way of capturing the imperfections in interbank markets that prevent banks from being able to perfectly target their end-of-day reserve balance.³ This uncertainty about the end-of-day balance creates a well-defined demand for interbank borrowing that responds smoothly to changes in interest rates and other parameters of the model.

The bank's balance sheet after the payment shock has been realized is

	Assets	Liabilities		
Loans	L	Deposits	$D-\varepsilon$	(4)
Bonds	В	Net interbank borrowing	$\Delta + \Delta_T$	(1)
Reserves	$R + \Delta + \Delta_T - \varepsilon$	Equity	E	

Depending on the size of payment shock, the bank may need to borrow from the central bank at the end of the day to meet its regulatory requirements. Such borrowing is done on an overnight basis and requires collateral. Let $X \ge 0$ denote the amount borrowed. If the bank pledges loans to the central bank as collateral and β represents the haircut applied by the central bank, its the end-of-day balance sheet is

As	sets	Liabilities		
Loans	L	Deposits	$D-\varepsilon$	
- hereof encumbered	$(1+\beta)X$	Net interbank borrowing	$\Delta + \Delta_T$	(5)
Bonds	В	Central bank borrowing	X	
Reserves	$R + \Delta + \Delta_T - \varepsilon + X$	Equity	E	

At this point, the day ends and payoffs are realized. The bank earns the interest rates r_L and r_B on its loans and bonds, respectively. It pays an interest rate r_D on customer deposits and pays (or

 $^{^{3}}$ See Afonso and Lagos (2012) for a model with explicit trading frictions in which a bank's end-of-day balance depends, in part, on the trading opportunities that arise.

earns) r and r_T on its interbank borrowing (or lending). The bank earns r_K on balances held at the central bank to meet its reserve requirement and r_R on any excess balances. In addition, the bank faces a penalty rate r_X (including the value of any associated stigma effects)⁴ for any funds borrowed from the central bank's lending facility.

3.2 Reserve Requirements

The bank faces a reserve requirement of the form

$$R + \Delta + \Delta_T - \varepsilon + X \ge K. \tag{6}$$

The left-hand side of this expression is the bank's reserve holdings at the end of the day, taking into account funds borrowed/lent in the interbank market, the payment shock, and borrowing from the central bank. The right-hand side is the requirement for the day, which we can think of as being determined by past values of items on the bank's balance sheet, such as deposits, but is a fixed number when the day begins. If the bank would violate the requirement (6) after the realization of the payment shock, it must borrow funds from the central bank to cover the deficiency.

Our simple model is sufficiently general to represent various types of operating frameworks used in practice to implement monetary policy. A standard corridor framework, for example, corresponds to situation where the interest rate r_R paid on excess reserves is set below the central bank's target rate, while a so-called floor system results if r_R is set equal to the target rate. The model could also represent an operational framework with no reserve requirement by setting K to zero, in which case (6) simply requires that a bank not end the period with an overdraft in its reserve account. For operating frameworks that allow reserve averaging, this simple model can be thought of as representing either the final day of a reserve maintenance period or the average values over the entire period.⁵

As borrowing from the central bank is costly, the bank will borrow the minimum amount needed

⁴See Ennis and Weinberg (2012) and Armantier et al. (2011) for discussions of the potential for stigma to be associated with borrowing from the central bank.

⁵See Ennis and Keister (2008) for a discussion of how this type of framework can be extended to include reserve averaging.

to meet its regulatory requirements. Let X_K denote the amount that must be borrowed to fulfill the reserve requirement in (6), that is,

$$X_K = \max\{K - R - \Delta - \Delta_T - \varepsilon, 0\}.$$
(7)

3.3 The Liquidity Coverage Ratio (LCR)

In the context of our model, the LCR of the representative bank at the opening of the interbank market can be written as

$$LCR = \frac{B+R}{\theta_D D} \ge 1 \tag{8}$$

Recall from (1) that the numerator of the ratio is the total value of the bank's high-quality liquid assets, which here simply equals bonds plus reserves.⁶ The denominator measures the 30-day net cash outflow assumed under the stress scenario, which here is simply the run-off of deposits at rate θ_D .⁷ At the end of the day, the bank's LCR becomes

$$LCR_{\varepsilon} = \frac{B + R + \Delta + \Delta_T - \varepsilon + X}{\theta_D (D - \varepsilon) + \theta_\Delta \Delta + \theta_X X} \ge 1,$$
(9)

where θ_{Δ} and θ_X are the run-off rates on overnight interbank loans and central banking borrowing, respectively. Terms loans expire outside the duration of the stress scenario and hence do not enter the denominator of the ratio. Under the LCR rules, the run-off factor for overnight interbank loans is 100% whereas the run-off rate on secured funding transactions with the central bank must be at least 25%. Hence, we set $\theta_{\Delta} = 1$ for our analysis, but treat θ_X as parameter that the authorities potentially can set higher than 25%.⁸

Let X_C denote the minimum amount the bank must borrow from the central bank to fulfill the

⁶For simplicity, we assume that reserves held to meet reserve requirements are included in the calculation of high-quality liquid assets. It is straightforward to modify the model to exclude these balances.

⁷The regulation considers two types of retail deposits: "stable" and "less stable". Stable retail deposits have a minimum run-off rate of 5% while less stable retail deposits have a minimum run-off rate of 10%. Stable retail deposits are those deposits that are fully covered by deposit insurance and where the depositors' relationship to the bank is deemed to make withdrawal sufficiently unlikely.

⁸In certain juridstictions, publicly-sponsored enterprises (e.g., government sponsored enterprices in the U.S.) are active in the interbank market. They receive more favorable treatment in the LCR calculations as unsecured funding from these entities has a run-off rate of only 75%. We study this special case in appendix [to be added].

LCR requirement in (9), that is,

$$X_C = \max\{\frac{C - R - \Delta_T + (1 - \theta_D)\varepsilon}{1 - \theta_X}, 0\},\tag{10}$$

where

$$C \equiv \theta_D D - B \tag{11}$$

represents the bank's shortfall (or surplus, if negative) of assets for LCR purposes before reserve holdings are taken into account. As described above, this value can be thought of as resulting from the bank's activities prior to the opening of the interbank market.

3.4 Borrowing from the central bank's lending facility

The bank will borrow the minimum amount needed from the central bank's lending facility to meet both its reserve and LCR requirements. Using (7) and (10), we have

$$X = \max\{X_K, X_C\} = \max\{K - R - \Delta - \Delta_T + \varepsilon, \frac{C - R - \Delta_T + (1 - \theta_D)\varepsilon}{1 - \theta_X}, 0\}.$$
 (12)

It will be useful to keep track of which of the two constraints determines the amount of borrowing from the standing facility for each realization of the payment shock ε . The bank must borrow from the central bank to satisfy the reserve requirement if and only if

$$\varepsilon > R + \Delta + \Delta_T - K \equiv \varepsilon_K(\Delta, \Delta_T). \tag{13}$$

Similarly, meeting the LCR requires use of the lending facility if and only if

$$\varepsilon > \frac{R + \Delta_T - C}{1 - \theta_D} \equiv \varepsilon_C(\Delta_T).$$
 (14)

If the bank must borrow to meet both requirements, the amount borrowed will be determined by the LCR if

$$\frac{C - R - \Delta_T + (1 - \theta_D)\varepsilon}{1 - \theta_X} > K + \varepsilon - R - \Delta - \Delta_T$$

and by the reserve requirement if this inequality is reversed. Straightforward algebra shows that this inequality holds and, hence, the bank's borrowing is determined by the need to meet the LCR requirement whenever

$$\varepsilon > \frac{(1-\theta_X)(K-\Delta) + \theta_X (R+\Delta_T) - C}{\theta_X - \theta_D} \equiv \hat{\varepsilon}(\Delta, \Delta_T).$$
(15)

Equations (13) – (15) define three critical values for the payment shock ε that characterize a bank's optimal borrowing from the central bank. Figure 1 illustrates the different possibilities under the assumption that $\theta_X > \theta_D$.⁹ In the left panel, $\varepsilon_K < \varepsilon_C$ holds and the bank will not need to borrow from the lending facility if $\varepsilon < \varepsilon_K^*$. If ε falls in the interval ($\varepsilon_K^*, \hat{\varepsilon}$), the bank will borrow just enough to meet the reserve requirement, which will lead it to oversatisfy the LCR requirement and will over-satisfy the reserve requirement.

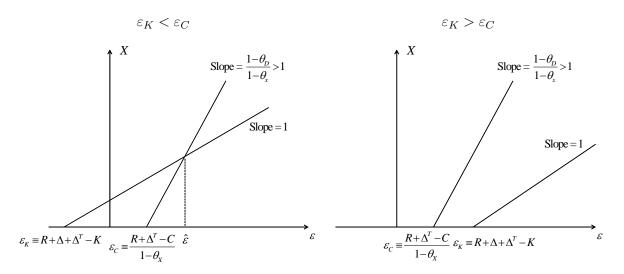


Figure 1: Borrowing from Central Bank Lending Facility $(\theta_x > \theta_D)$ 10

In the right panel of the figure, $\varepsilon_K \ge \varepsilon_C$ holds and the amount of borrowing is always determined by the LCR requirement. When ε is below ε_C , the bank does not borrow from the lending facility.

⁹According to the minimum standard set forth in the LCR rules text $\theta_D > 5\%$ or 10% for "stable" or "less stable" deposits, respectively (see footnote 5) while $\theta_x > 25\%$.

When ε is above this value, the bank borrows just enough satisfy the LCR requirement. Notice that the reserve requirement never binds in this panel; for every value of the payment shock ε , the bank ends up holding more than the required quantity of reserves.

4 Equilibrium Interest Rates

In this section, we show how the equilibrium interest rates in the overnight and term markets depend on the quantity of reserves supplied by the central bank and the LCR position of the banking system. We begin by deriving the representative bank's demand for funds in each of these markets.

4.1 The demand for funds

The bank's profit for the period can be written as

$$\pi(\Delta, \Delta_T; \varepsilon) = r_L L + r_B B - r_D (D - \varepsilon) + r_K K + r_R \max \{R + \Delta + \Delta_T + X - \varepsilon - K, 0\} (16)$$
$$-r\Delta - r_T \Delta_T - r_X X.$$

The first line in (16) represents interest earned on loans and bonds minus the interest cost of deposits, plus the interest earned on required and excess reserves. The second line reflects the cost (or revenue) associated with net interbank borrowings and the cost of any borrowing from the central bank. The bank will choose its interbank borrowing activity (Δ, Δ_T) to maximize the expected value of the expression in (16). Using (12) and $E[\varepsilon] = 0$, and rearranging terms, we can write this expected value as

$$E[\pi(\Delta, \Delta_T)] = r_L L + r_B B - r_D D + r_K K - r\Delta - r_T \Delta_T + r_R (R + \Delta + \Delta_T - K)$$
(17)
$$-(r_X - r_R) E\left[\max\left\{K - R - \Delta - \Delta_T + \varepsilon, \frac{C - R - \Delta_T + (1 - \theta_D)\varepsilon}{1 - \theta_X}, 0\right\}\right].$$

Dropping the terms that do not depend on the bank's choices of Δ and Δ_T , the maximization problem can be written as

$$\max_{(\Delta,\Delta_T)} -r\Delta - r_T \Delta_T + r_R \left(R + \Delta + \Delta_T - K \right)$$

$$- (r_X - r_R) \left\{ \begin{array}{c} \mathbf{1}(\varepsilon_K < \varepsilon_C) \int_{\varepsilon_K}^{\hat{\varepsilon}} \left(K - R - \Delta - \Delta_T + \varepsilon \right) g\left(\varepsilon\right) d\varepsilon \\ + \int_{\max\{\hat{\varepsilon},\varepsilon_C\}}^{\infty} \frac{C - R - \Delta_T + (1 - \theta_D)\varepsilon}{1 - \theta_X} g\left(\varepsilon\right) d\varepsilon \end{array} \right\},$$
(18)

where g is the density function associated with the distribution G of the payment shock. The indicator function $\mathbf{1}(\cdot)$ takes value one if the expression in parentheses is true and zero otherwise.

The solution to this problem is characterized in the following proposition, a proof of which is provided in the appendix.

Proposition 1 If $r > r_R$, the bank will choose (Δ, Δ_T) to satisfy

$$r = r_R + (r_X - r_R) \left(G[\hat{\varepsilon}(\Delta, \Delta_T)] - G[\varepsilon_K(\Delta, \Delta_T)] \right)$$
(19)

and

$$r_T = r + \frac{(r_X - r_R)}{1 - \theta_X} \left(1 - G\left[\max\left\{\hat{\varepsilon}(\Delta, \Delta_T), \varepsilon_C(\Delta_T)\right\}\right]\right)$$
(20)

If $r = r_R$, the bank will choose Δ_T to satisfy (20) and will be indifferent between any values of Δ satisfying

$$\Delta \ge \frac{(1-\theta_X) K + \theta_X (R + \Delta_T) - C}{(1-\theta_X)}.$$
(21)

Equation (19) shows that the marginal value the bank places on an additional unit of overnight borrowing, r, has two components. The first is simply the interest rate it earns by holding the funds on deposit at the central bank, r_R . The second component is proportional to the probability that ε will fall between ε_K and $\hat{\varepsilon}$, in which case the amount it borrows from central bank will be determined by the need to meet its reserve requirement. In these states, an additional unit of overnight borrowing reduces the amount that must be borrowed from the central bank by one unit, saving the bank r_X . Equation (20) shows that the marginal value of funds in the term interbank market is equal to the value of overnight funds r plus a term premium. This term premium is proportional to the probability that ε will be large enough for the amount the bank borrows from the central bank to be determined by its need to meet the LCR requirement. In these states, an additional unit of term borrowing reduces the amount that must be borrowed from the central bank, whereas an additional unit of overnight borrowing does not.

4.2 Equilibrium

Because banks are identical when the interbank market meets, there will be no trade in equilibrium and the market-clearing condition can be written as¹¹

$$\Delta = \Delta_T = 0. \tag{22}$$

By imposing these equilibrium conditions in the demand functions from Proposition 1, we can derive the equilibrium interest rates (r^*, r_T^*) as functions of the elements of the initial balance sheet (2), including the quantity of reserves R supplied by the central bank and the bank's liquidity deficit (net of reserves) C. In equilibrium, the cutoff values (13) – (15) for the payment shock ε are

$$\varepsilon_K^* \equiv \varepsilon_K(0,0) = R - K,\tag{23}$$

$$\varepsilon_C^* \equiv \varepsilon_C(0) = \frac{R - C}{1 - \theta_D},\tag{24}$$

and

$$\widehat{\varepsilon}^* \equiv \widehat{\varepsilon}(0,0) = \frac{(1-\theta_X)K + \theta_X R - C}{\theta_X - \theta_D}.$$
(25)

Substituting these expressions into the demand functions (19) and (20) yields the equilibrium pricing relationships, which are presented in the following proposition.

Proposition 2 Given the initial balance sheet of the banking system in (2), the equilibrium interest

¹¹In a setting with heterogeneous banks, indexed by *i*, the equilibrium condition requires that net interbank borrowing be zero, that is, $\Sigma_i \Delta_i = \Sigma_i \Delta_{Ti} = 0$.

rates in the interbank markets are

$$r^* = r_R + (r_X - r_R) \max\{G[\hat{\varepsilon}^*] - G[\varepsilon_K^*], 0\}$$
(26)

and

$$r_T^* = r^* + \frac{(r_X - r_R)}{1 - \theta_X} (1 - G[\max\{\widehat{\varepsilon}^*, \varepsilon_C^*\}]).$$

$$(27)$$

This result establishes how the supply of reserves R and other elements of the balance sheet (2) affect equilibrium interest rates. Equation (26) shows that the overnight interest rate equals the rate paid on excess reserves r_R plus a premium that reflects the marginal value of overnight funds in avoiding a potential deficiency in the reserve requirement. To understand this relationship, suppose a bank borrows an additional dollar in the overnight market and holds this dollar in its reserve account. The value generated by this dollar of funds will depend on the realized value of the bank's payment shock ε . If ε is below ε_K , the bank will satisfy its regulatory requirements without borrowing from the central bank and the extra dollar will simply earn the interest rate on excess reserves r_R . If ε is above $\hat{\varepsilon}$, the amount the bank borrows from the central bank will be determined by its need to satisfy the LCR requirement (see the left panel of Figure 1), which is not affected by this extra dollar of overnight borrowing. In this case, the dollar will again be held as excess reserves and earn the rate r_R . If, however, ε falls in between ε_K and $\hat{\varepsilon}$, the bank will borrow just enough from the central bank to meet its reserve requirement. In this case, the additional dollar of reserves will allow the bank to borrow one dollar less from the central bank, saving it the penalty rate r_X . The expected value of an additional dollar of overnight funding is, therefore,

$$r_R\left[\operatorname{prob}\left(\varepsilon \le \varepsilon_K\right) + \operatorname{prob}\left(\varepsilon \ge \widehat{\varepsilon}\right)\right] + r_X\left[\operatorname{prob}\left(\varepsilon_K < \varepsilon < \widehat{\varepsilon}\right)\right] = r_R + (r_X - r_R)\operatorname{prob}\left(\varepsilon_K \le \varepsilon \le \widehat{\varepsilon}\right)$$
(28)

Using the distribution function G to determine these probabilities and rearranging terms, this expression is equivalent to the right-hand side of (26) and thus determines the equilibrium interest rate in the overnight market.

Equation (27) shows how the equilibrium term premium $r_T^* - r^* \equiv p^*$ reflects the marginal value of term borrowing in avoiding a potential deficiency in the LCR requirement. If a bank borrows an additional dollar of funds in the term market, the value of this dollar will again depend on the realized value of the bank's payment shock. If ε is below both ε_K and ε_C , the bank has no need to borrow from the central bank and the extra dollar will earn the interest rate on excess reserves r_R . If $\varepsilon_R < \varepsilon_C$ (as in the left panel of Figure 1) and ε falls falls in between ε_K and $\hat{\varepsilon}$, the extra dollar of term borrowing will lower the amount the bank needs to borrow from the central bank to meet its reserve requirement (just like an extra dollar of overnight borrowing would), saving the bank the penalty rate r_X . The additional benefit of term borrowing comes if ε is larger than both ε_C and $\hat{\varepsilon}$. In this case, the extra dollar decreases the amount the bank needs to borrow from the central bank to meet its LCR requirement by $(1 - \theta_X)^{-1} > 1$ dollars. Notice that because loans from the central bank have a runoff rate of θ_X for LCR purposes, one dollar of term borrowing saves the bank from having to borrow more than a dollar from the central bank in this case. The expected value of an additional dollar of term funding can, therefore, be written as

$$r_{R}\left[\operatorname{prob}\left(\varepsilon \leq \min\left\{\varepsilon_{K}, \varepsilon_{C}\right\}\right)\right] + r_{X}\left[\operatorname{prob}\left(\varepsilon_{K} < \varepsilon < \widehat{\varepsilon}\right)\right] + \frac{r_{X}}{1 - \theta_{X}}\left[\operatorname{prob}\left(\varepsilon \geq \max\left\{\widehat{\varepsilon}, \varepsilon_{C}\right\}\right)\right]$$
(29)

After some manipulation, this expression can be show to be equivalent to the right-hand side of (27) and thus represents the equilibrium interest rate in the term interbank market.

If the likelihood of the bank needing to borrow from the central bank for LCR purposes is very small, our setup reduces to a version of the standard Poole (1968) model. This fact can be seen by setting the LCR deficiency C well below zero, in which case ε_C and $\hat{\varepsilon}$ both become very large. As the probability that ε will be larger than $\hat{\varepsilon}$ falls to zero, the interest rate in the overnight market will be driven solely by concerns related to reserve requirements. The term premium will also disappear, since the only advantage of term borrowing in this model is that it raises a bank's LCR. These results are summarized in the following corollary.

Corollary 1 (Poole (1968) Model) In the limit as C approaches $-\infty$, the interest rate in the

overnight interbank market is given by

$$r^* = r_R + (r_X - r_R)(1 - G[\varepsilon_K^*])$$

and the term premium is zero, that is, $r_T^* = r^*$.

Equations (26) and (27) can also be used to determine bounds for the equilibrium interest rate r^* and r_T^* . Since G is a probability distribution function, its value is always between zero and one. It follows immediately from (26) that the overnight interest rate must lie in the interval $[r_R, r_X]$. In other words, as is standard, the overnight rate will lie in the corridor formed by the interest rate on excess reserves r_R and the cost of borrowing from the central bank r_X , where the latter includes the value of any stigma-related concerns. It follows from equation (27) that the term premium is always non-negative and can be as large as $(r_X - r_R) / (1 - \theta_X)$. Note that the term interest rate r_T can thus rise above the upper bound of the corridor r_X , since term interbank borrowing lowers an LCR deficiency dollar-for-dollar, while borrowing from the central bank has a runoff rate of θ_X .

Corollary 2 The equilibrium interest rate in the overnight interbank market satisfies

$$r_R \le r^* \le r_X,$$

and the equilibrium term premium satisfies

$$0 \le r_T^* - r^* \le \frac{(r_X - r_R)}{1 - \theta_X}.$$

In the absence of an LCR requirement, the overnight interest rate will equal the midpoint of the corridor when reserve conditions are "balanced" in the sense that the supply of reserves R is exactly equal to the total requirement K (see Whitesell, 2006). Suppose that "liquidity conditions" are also balanced, so that R = K = C, meaning that there are just enough reserves and bonds in the banking system to allow both the reserve requirement and the LCR to be met. The next corollary shows that, in this case, the overnight interest rate is equal to the floor of the corridor and the term premium equals half of its maximum possible value. **Corollary 3** Suppose liquidity conditions are "balanced" in the sense that R = K = C. Then the equilibrium interest rate in the overnight interbank market is equal to the floor of the corridor, $r^* = r_R$, and the equilibrium term premium is

$$r_T^* - r^* = \frac{1}{2} \frac{(r_X - r_R)}{1 - \theta_X}.$$

Taken together, Corollaries 1 - 3 illustrate how the introduction of an LCR requirement can radically alter the relationship between the quantity of reserves and equilibrium interest rates. Without the LCR, Corollary 1 shows that when reserve supply equals the total requirement K, the overnight trade will trade at the midpoint of the corridor range, which is often the central bank's target value. Corollary 3 shows that using the same modus operandi when there are just enough liquid assets in the banking system to satisfy banks' LCR requirements pushes the overnight rate to the floor of the corridor and makes the very short end of the yield curve steeper.

4.3 Examples

Before moving to the study of central bank operations, we use a pair of numerical examples to illustrate the features of our model and the determinants of the equilibrium interest rates (r^*, r_T^*) . We first look at the limiting case where $C \to -\infty$, which implies that the LCR requirement is always satisfied. Corollary 1 establishes that the term premium in this case is zero and, hence, only the overnight interest rate r^* needs to be determined.

Example 4 Assume $C \to -\infty$ and let $r_R = 2\%$, $r_X = 4\%$, K = 0, and $\varepsilon \sim N(0, 1)$. The figure in the left-hand panel below depicts the bank's demand for overnight borrowing Δ for three different levels of reserve supply R. These curves are drawn with the level of term borrowing Δ_T set to zero, so that (13) implies

$$\varepsilon_K(\Delta, 0) = R + \Delta$$

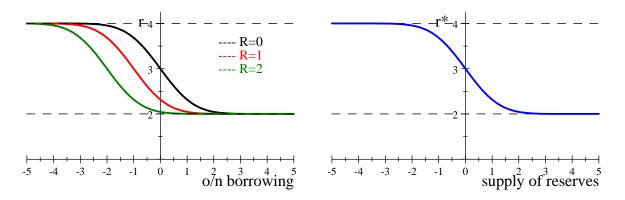
and the demand curve (19) depicted in this panel is

$$r = 2\% + 2\%(1 - \Phi[R + \Delta]).$$

The equilibrium interest rate is determined by setting $\Delta = 0$ for each possible value of R and finding the corresponding value of r^* on the vertical axis. This relationship, which can be written as

$$r^*(R) = 2\% + 2\%(1 - \Phi[R]),$$

is depicted in the right-hand panel below.



This first example illustrates that when banks are not concerned about meeting the LCR requirement, both the demand for interbank borrowing and the relationship between the level of reserves and the equilibrium interest rate in our model are completely standard.

Next, we present an example in which the LCR may be violated and a term premium emerges in equilibrium. We focus on how the demand for interbank borrowing and the equilibrium interest rates depend on the size of the LCR deficit C. We use similar values for the other parameters as in the previous example.

Example 5 Let $r_R = 2\%$, $r_X = 4\%$, R = K = 0, $\theta_X = .25$, $\theta_D = .05$ and $\varepsilon \sim N(0, 1)$. Then (13) becomes $\varepsilon_K(\Delta, \Delta_T) = \Delta + \Delta_T$ and (15) becomes $\hat{\varepsilon}(\Delta, \Delta_T) = 1.25\Delta_T - 3.75\Delta - 5C$. Substituting these expressions into (19) and (20) yields

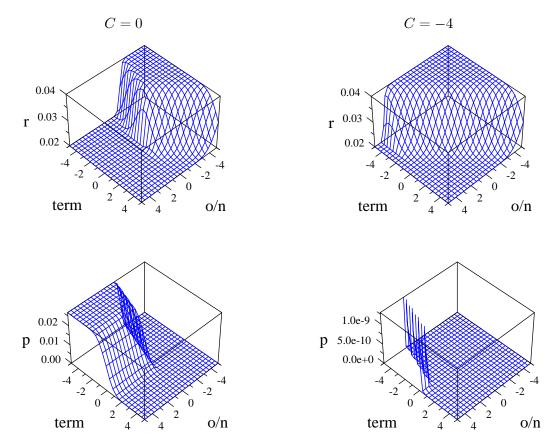
$$r = 2\% + 2\% \max\{\Phi[1.25\Delta_T - 3.75\Delta - 5C] - \Phi[\Delta + \Delta_T], 0\}$$

and

$$p \equiv r_T - r = \frac{2\%}{1 - .25} \{ 1 - \Phi[\max\{1.25\Delta_T - 3.75\Delta - 5C, \frac{\Delta_T - C}{1 - .05} \}]$$

The figure below corresponds to the left-hand panel in the previous example; it show the bank's demand for term and overnight borrowing for different levels of the overnight interest rate r and the term premium p. The two panels on the left correspond to a configuration of parameters in which the bank is somewhat likely to face an LCR deficit, while the two panels on the right-hand side represent a situation where the LCR is more likely to be satisfied. Notice the different scales

for the vertical axis in the bottom panels.



The equilibrium interest rates are given by

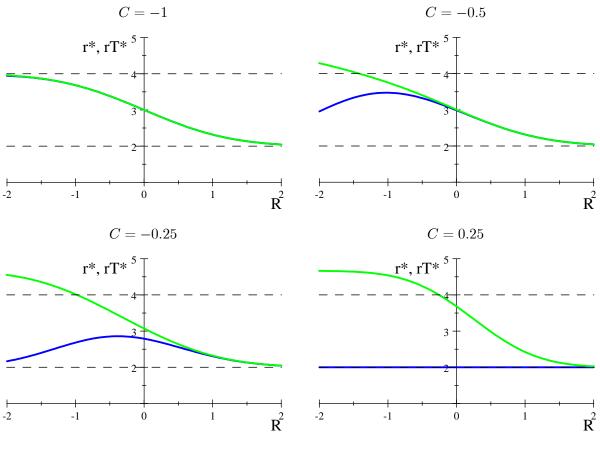
$$r^* = 2\% + 2\% \max(\Phi[-5C] - \Phi[0], 0)$$

and

$$r_T^* = r^* + \frac{2\%}{1 - .25} (1 - \Phi(\max(-5C, \frac{0 - C}{1 - .25}))).$$

The next figure presents the equilibrium relationship between these rates and the level of reserves R. When C is sufficiently small, as in the upper-left panel, the LCR has a negligible effect on equilibrium prices: the overnight rate is essentially unchanged from the earlier example and the term premium is zero. As C become progressively larger, the LCR requirement becomes more likely to bind and, hence, has a larger impact on prices. A term premium emerges, first for sufficiently low values of R and then eventually for higher values of R as well. For C = 0.25, the overnight interest rate remains at the floor of the corridor for all values of R because in the process of satisfying the LCR requirement, banks will over-satisfy their reserve requirement, as in the right-hand panel of





- overnight rate, - term rate

This second example demonstrates how the relationship between the quantity of reserves and interbank interest rates depends critically on the size of banks' liquidity deficit net of reserves, C. In varying the level of reserves R, each panel in the figure above holds C fixed at the specified level. However, most types of open market operations that central banks use to change the supply of reserves will affect bank balance sheets in a way that changes the liquidity deficit C, adding a further complication. In the next section, we introduce explicit open market operations to the model and study how the central bank's ability to control market rates depends on the form these operations take.

5 Open market operations and the LCR

In this section, we investigate the effect of open market operations on equilibrium interest rates in the presence of an LCR.¹² We show that it is not only the size of the OMO that matters in this setting, but also the way in which the OMOs are conducted. This is an important insight, as OMOs differ both within and across central banks along a number of dimensions.¹³ These dimensions include *outright* purchases (or sales) versus *reverse* operations, *asset eligible* for purchase or as collateral, *haircuts* on collateral, *counterparties* to the operations (banks versus non-banks) and *maturity of operations* (overnight, two weeks, or longer).

In the standard model, different types of OMOs affect interest rates in a similar fashion. Adding or subtracting reserves loosen or tightens the reserve requirement constraint dollar for dollar. The other changes to bank balance sheets induced by the operation do not influence the outcome. When an LCR requirement is in place, however, these other balance sheet changes *do* matter as they also impact banks' ability to meet this requirement.

5.1 Adding OMOs to the model

We now allow the central bank to engage in open market operations after the balance sheet items in (2) have been determined, but before interbank markets open.¹⁴ Let z denote the quantity of additional reserves created by the operation. Our interest in this section is in understanding how the interbank interest rates r and r_T vary with z. In choosing how to structure the open market operation, the central bank faces a number of choices along the dimensions listed above. We start by looking at outright purchases and then turn to reverse operations.

 $^{^{12}}$ Open market operations (OMOs) are monetary policy operations conducted at the discretion of the central bank. Originally, the expression referred to operations in the open market (i.e. secondary or inter-bank market) where the central bank acted as a normal, possibly anonymous, participant, for instance by buying or selling Treasury securities (Bindseil 2004). Later the expression also began to cover so-called reverse operations where the central bank undoes the initial operation at a later stage.

¹³For example, the Federal Reserve distinguishes between *temporary* and *permanent* OMOs. Temporary OMOs involve repurchase and reverse repurchase agreements that are designed to temporarily add to or subtract from the total supply of reserves in the banking system. Permanent OMOs involve the buying and selling of securities outright to permanently add or subtract reserves. The ECB, in contrast, relies to a large extent on revolving reserve operations of various maturities.

¹⁴As in Bartolini, Bertola and Prati (2002), a random shock reflecting uncertainty with regards to net autonomous factors could easily be added after the open market operation.

5.1.1 Outright purchases

In our simple framework, if the central bank chooses to add the reserves via outright purchases, it can either buy the assets from the bank or from the non-bank sector. Moreover, it can either buy bonds, i.e., HQLA, or (pools of) loans, i.e., non-HQLA. We consider the various cases in turn.

Purchases from the banking system If the central bank purchases bonds from the representative bank, the balance sheet (2) and the LCR (8) adjust as follows:

Asset	ts	Liabiliti	\mathbf{es}		
Loans	L	Deposits	D	$\Rightarrow LCR_1 = \frac{B - z + R + z}{\theta_D D} = LCR_0$	(30)
Bonds	B-z			$\Rightarrow LC n_1 - \frac{\theta_D D}{\theta_D D} = LC n_0$	(50)
Reserves	R+z	Equity	E		

The size of the bank's balance sheet is unchanged as the purchases only alter the composition of the asset side. Moreover, the stock of HQLA held by the bank is unchanged as well, since one type of Level 1 asset (reserves) is replacing another (bonds). Consequently, the bank's LCR is unaffected.

If the central bank buys non-HQLA assets from the banking sector, in contrast, the LCR increases as shown in the chart below. As in the previous case, the operation leaves the size of the bank's balance sheet unchanged but alters the composition of its assets. In this case, though, the stock of HQLA increases and, hence, the bank's LCR increases.

AssetsLiabilitiesLoans
$$L-z$$
Deposits D Bonds B $\rightarrow LCR_1 = \frac{B+R+z}{\theta_D D} > LCR_0$ (31)Reserves $R+z$ Equity E

Purchases from the non-bank sector If the central bank buys assets from the non-bank sector, the LCR increases as well. The proceeds from the sale are credited to the banking system as reserves and the non-bank sector receives a claim - equal to the sales amount - on the banking system in form of a deposit. Consequently, the balance sheet of the banking system expands. The

stock of high-quality liquid assets grows by the amount of the reserves, but this increase is partly offset in the LCR calculation by a rise in the net cash outflow due to the additional deposits. If the newly-created deposits have a runoff rate θ_D that is fairly small, the increase in the LCR could be significant. Notice that, in this case, the type of assets purchased by the central bank does not matter for the LCR.

Asse	ts	Liabil	ities		
Loans	L	Deposits	D+z	$\Rightarrow LCR_1 = \frac{B - R + z}{\theta_D(D + z)} > LCR_0$	(32)
Bonds	B			$\Rightarrow LOR_1 = \theta_D(D+z) > LOR_0$	(02)
Reserves	R+z	Equity	E		

5.1.2 Reverse operations

Things are slightly more complex if the central bank uses reverse operations. As with outright purchases, the impact on the LCR depends on the type of assets involved (in this case as collateral) and the counterparty to the transactions. In addition, the impact of the operation on the LCR depends on the duration and on the collateral haircuts applied by the central bank.

Reverse operations with the banking system If the central bank conducts reverse operations with the representative bank, the LCR decreases if the collateral is HQLA and the central bank applies a haircut. As shown in the chart below, the stock of high-quality liquid assets used in the LCR calculation only includes assets that are unencumbered.

Asset	s	Liabiliti	\mathbf{es}		
Loans	L	Deposits	D		
Bonds	В			$\Rightarrow LCR_1 = \frac{B + R - \alpha z}{\theta_D D} \lesssim LCR_0$	(33)
- encumbered	$(1+\alpha)z$				
Reserves	R+z	Equity	E		

Here α denotes the haircut on high-quality liquid assets set by the central bank. In this operation, one unit of reserves is exchanged for $1 + \alpha$ units of bonds, shrinking the stock of HQLA available to the bank. However, the haircuts applied to high-quality liquid assets tend to be small and thus the impact of this type of operation on the LCR is likely to be small as well.

If the central bank instead conducts reverse operations against non-HQLA, the LCR of the banking system increases. The increase is larger if the (remaining) duration, d, of the operation is longer than 30 days. If it is less than 30 days, then an amount is added to the net cash outflow in the denominator of the ratio according to the rules of the regulation.

AssetsLiabilitiesLoansLDepositsD-encumbered
$$(1 + \beta)z$$
 $\Rightarrow LCR = \begin{cases} \frac{B+R+z}{\theta_D D + \theta_x z}, & d \leq 30\\ \frac{B+R+z}{\theta_D D}, & d > 30 \end{cases} > LCR_0$ (34)BondsB $=$ Reserves $R+z$ EquityE

Here β is the haircut applied by the central bank on non high-quality liquid assets. Note that the LCR impact of reverse operations longer than 30 days is the same as for outright purchases of non-HQLA.

Reverse operations with the non-bank sector Reverse operations with the non-bank sector (regardless of duration) increase the LCR and have the same effect as outright purchases from the non-bank sector.

Table 1 summarizes the impact of open market operations on the LCR of the banking system.

		Bank	Non-bank	
Type	Term	HQLA	$\operatorname{non-HQLA}$	
Outright	∞	$\frac{B+R}{\theta_D D} = LCR_0$	$\frac{B+R+z}{\theta_D D}$	$\frac{B+R+z}{\theta_D(D+z)}$
Reverse ops.	$\leq 30 \text{ days}$	$\frac{B+R-\alpha z}{\theta_D D} \approx LCR_0$	$\frac{B+R+z}{\theta_D D+\theta_X z}$	$\frac{B+R+z}{\theta_D(D+z)}$
	> 30 days	$\frac{B+R-\alpha z}{\theta_D D} \approx LCR_0$	$\frac{B+R+z}{\theta_D D}$	$\frac{B+R+z}{\theta_D(D+z)}$

Table 1: The Impact of Open Market Operations on the LCR

Assuming that the haircut applied on HQLA are small enough to have a negligible effect on the

LCR, we have four distinct cases to study: (i) Purchases of (or reverse operations against) HQLA from banks, (ii) Purchases of (or longer-term reverse operations against) non-HQLA from banks, (iii) short term reverse operations against non-HQLA with banks, and (iv) OMOs with the nonbank sector (i.e. entities not subject to reserve requirements and the LCR). In the next subsection, we study the impact of OMOs on market interest rates for each of these four cases.

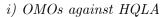
5.2 Equilibrium Rates

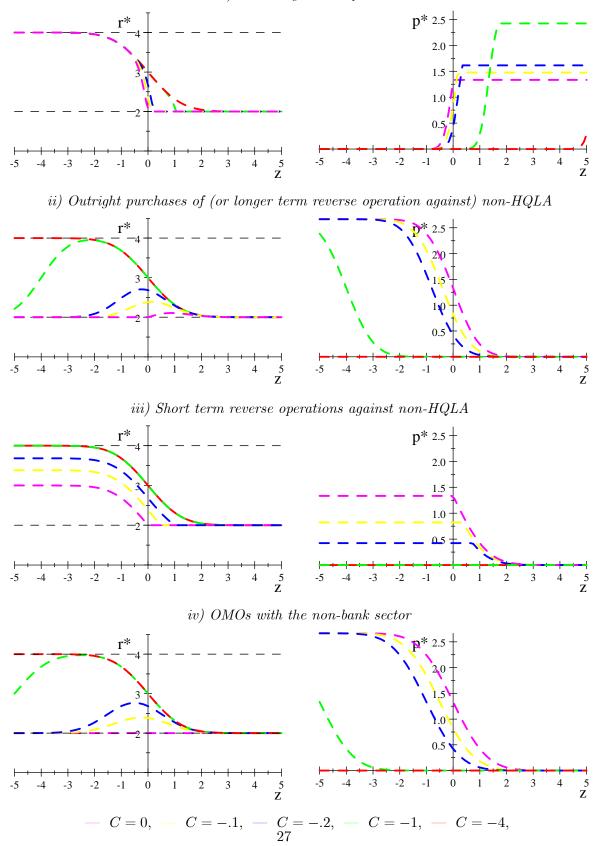
Above, we showed how the LCR is impacted differently depending on the way an open market operation is structured. Now, we use this setup to study how different types of OMOs affect equilibrium interest rates within a corridor system when an LCR requirement is present. Again, we turn to numerical examples.

Example 6 Assume that $r^R = 2\%$, $r^K = 4\%$, $R_0 = K = 0$, $\theta_x = .25$, $\theta_D = .05$ and $\varepsilon \sim N(0, 1)$. Let $R_1 = z$ and C_1 denote the amount of reserves and the LCR shortfall ex reserves, respectively, after the central bank has conducted OMOs of size z. From (23), (24) and (25) we have that the new cut-off values for the payment shock ε are

Case	-	$\varepsilon_K^*(R_1)$	$\varepsilon_C^*(R_1, C_1)$	$\hat{arepsilon}^*(R_1,C_1)$
i)	$\theta_D D - (B - z)$	z	$\frac{-C_0}{1-\theta_D}$	$-rac{(1- heta_x)}{ heta_X- heta_D}z-rac{C_0}{ heta_X- heta_D}$
ii)	$\theta_D D - B$	z	$\frac{z-C_0}{1-\theta_D}$	$\frac{\theta_x}{\theta_X - \theta_D} z - \frac{C_0}{\theta_X - \theta_D}$
iii)	$\theta_D D + \theta_X z - B$	z	$\frac{(1- heta_x)z-C_0}{1- heta_D}$	$-\frac{C_0}{\theta_X-\theta_D}$
iv)	$\theta_D(D+z) - B$	z	$z - \frac{C_0}{1 - \theta_D}$	$z - \frac{C_0}{\theta_X - \theta_D}$

where i) - iv) are the four distinct types of OMOs identified above. Substituting these cut-off values into (26) and (27) yields a relationship between the size of the operation z and the equilibrium interest rates. The plots below show these relationships, with the overnight rate r^* depicted in the left-hand panels and the term premium $p^* \equiv r^* - r_T^*$ in the right-hand panels, for different values of the LCR deficiency C.





In all four cases, when banks have a large LCR surplus (i.e., C = -4), the effect of open market operations is exactly the same as in a standard model with no LCR requirement. In particular, the overnight interest rate is the same, monotone function of total reserves R + z in all four cases (matching the right-hand panel of the figure in Example 4) and the term premium is zero. In these situations, the standard approach of setting the total supply of reserves equal to the total requirement will ensure that both the overnight and the term interest rate equal the midpoint of the corridor.

For smaller values of the LCR surplus, where the requirement is more likely to bind in some states, the four cases differ is some respects. In each case, the overnight interest rate is pushed lower for the reasons discussed above. As it becomes more likely that the bank will need to borrow from the central bank at the end of the day to meet its LCR requirement, it becomes less likely that the bank will face a deficiency in its reserve requirement. As a result, the value of overnight funding, which can mitigate a deficiency in the reserve requirement but not in the LCR, decreases. The relationship between the size of the open market operation z and the equilibrium interest rate r differs across cases, with the relationship remaining monotone in cases (i) and (ii) but becoming non-monotone in cases (ii) and (iv). These differences arise because the different types of open market operations have different effects on the stock of HQLA held by banks and, hence, on the likelihood that the LCR will bind in some states. For example, when the central bank drains reserves (i.e., sets z < 0) by selling non-HQLA (case (*ii*)), it lowers the stock of HQLA and makes the LCR requirement more likely to bind. For this reason, the overnight interest rate may actually fall as z becomes more negative and, for some values of C, may remain at or near the floor of the corridor for all values of z. In these situations, the central bank may be unable to achieve a target interest rate in the overnight market using this type of operation. A similar pattern arises in case (iv), where the central bank drains reserves by selling assets to the non-bank sector and again lowers the LCR of the banking system in the process. In both of these cases, the term premium becomes substantial when z is negative, as the likelihood that the LCR will bind leads banks to place a high value on term funding.

6 Conclusions

The introduction of the liquidity coverage ratio will influence banks' demand for funds in the interbank market. Central banks that conduct monetary policy by setting a target for the interest rate in this market will, therefore, likely need to adapt their procedures to take this change into account. In this paper, we analyze how the introduction of an LCR affects the process of monetary policy implementation in the context of a simple, well-known model of banks' reserve management. This analysis points to three basic conclusions. First, if the LCR is far from binding, it will have little or no impact on the process of monetary policy implementation. Second, if, however, the LCR is a binding constraint in some states then the central bank may have difficulty controlling the overnight interest rate using open market operations. Finally, the LCR will tend to increase the steepness of the very short end of the yield curve.

There are many potential remedies that can alleviate the concerns we raise. In certain jurisdictions, it might be advantageous for the central bank to target the term rate rather than the overnight rate. The Swiss National Bank, for example, already implements its monetary policy in part by fixing a target range for the three-month Swiss franc Libor rather than the overnight rate. Another option is for the central bank to stand ready to provide the banking system with a form of high-quality liquid assets other than reserves. Inspiration in terms of designing such facilities can be taken from the Bank of England's Discount Window and the Federal Reserve's Term Securities Lending Facility, which allow participants to borrow, for a fee, gilts or treasuries against a wider range of potentially less liquid collateral. Another way to supply high-quality liquid assets is to sell call options on reserves as, for example, offered by the Federal Reserve in connection with the Y2K concerns in late 1999. In fact, the regulation already provides jurisdictions that, in general, have insufficient amounts of high-quality liquid assets the possibility of providing contractual committed liquidity facilities for a fee that counts towards the stock of liquid assets.

Changes to the regulation itself can also be envisioned. Just as central banks use reserve averaging to smooth out the impact of shocks to the reserve position of the banking system, some form of LCR averaging can be employed. This would reduce the need for otherwise sound banks to resort to the central bank lending facility for LCR purposes. An even more radical solution would be to eliminate reserves from the calculation of the LCR.

In this paper, we focus on the case where banks can borrow overnight from the central bank at

the end of the day against illiquid collateral and can use the acquired reserves to raise their LCR. In some jurisdictions, however, such borrowing is only available against high-quality and liquid collateral (see Appendix B). In these cases, the central bank's ability to control the overnight interest rate would be unaffected by an LCR requirement in our model, but the term premium would still be determined by LCR considerations. In an expanded model that accounts for how banks arrive at their initial balances sheets, the lack of a marginal supplier of HQLA at the end of day will induce banks to hold more high-quality liquid assets. In other words, in such cases one would expect banks to hold an additional buffer of HQLA, which could potentially affect other components of the monetary policy transmission mechanism. For example, at its June 2012 meeting, the Bank of England's Monetary Policy Committee (MPC) "discussed the possibility that regulatory liquidity requirements might be increasing the demand for reserves, attenuating the impact on the economy of the MPC's asset purchase programme and the associated increase in the supply of reserves."¹⁵ Understanding how the LCR affects these aspects of monetary policy implementation is an interesting question for future research.

7 Appendix

A Proofs of Propositions

Proposition 1: If $r > r_R$, the bank will choose (Δ, Δ_T) to satisfy (19) and (20). If $r_T = r$, the bank will choose Δ_T to satisfy (20) and will be indifferent between any values of Δ satisfying (21).

Proof: To deal with the indicator function in the objective function (18), we examine the firstorder conditions in the regions where $\varepsilon_K \geq \varepsilon_C$ and $\varepsilon_K < \varepsilon_C$ hold separately. If $\varepsilon_K \geq \varepsilon_C$ holds, the indicator function is zero and we also have max $\{\hat{\varepsilon}, \varepsilon_C\} = \varepsilon_C$. The marginal benefit of overnight and term loans in this case are, therefore, given by

$$\frac{\partial E[\pi]}{\partial \Delta} = -r + r_R \tag{35}$$

¹⁵www.bankofengland.co.uk/publications/Documents/records/fpc/pdf/2012/record1207.pdf page 9.

and

$$\frac{\partial E[\pi]}{\partial \Delta_T} = -r_T + r_R + \frac{(r_X - r_R)}{1 - \theta_X} \int_{\varepsilon_C}^{\infty} g(\varepsilon) \, d\varepsilon.$$
(36)

In the region where $\varepsilon_K < \varepsilon_C$ holds, the value of the indicator function is one and we have

$$\frac{\partial E[\pi]}{\partial \Delta} = -r + r_R + (r_X - r_R) \int_{\varepsilon_K}^{\hat{\varepsilon}} g(\varepsilon) \, d\varepsilon \tag{37}$$

and

$$\frac{\partial E[\pi]}{\partial \Delta_T} = -r_T + r_R + (r_X - r_R) \left\{ \int_{\varepsilon_K}^{\hat{\varepsilon}} g(\varepsilon) \, d\varepsilon + \frac{1}{1 - \theta_X} \int_{\hat{\varepsilon}}^{\infty} g(\varepsilon) \, d\varepsilon \right\}.$$
(38)

Suppose that $r > r_R$ holds. Expression (35) shows that, in this case, expected profit is strictly decreasing in Δ in the region where $\varepsilon_K > \varepsilon_C$ holds and, hence, the solution must satisfy $\varepsilon_K < \varepsilon_C$. This solution is characterized by setting the derivatives (37) and (38) to zero. Solving these two equations for the interest rates r and r_T yields

$$r = r_R + (r_X - r_R)(G[\hat{\varepsilon}(\Delta, \Delta_T)] - G[\varepsilon_K(\Delta, \Delta_T)])$$
(39)

and

$$r_T = r + \frac{(r_X - r_R)}{1 - \theta_X} (1 - G[\hat{\varepsilon}(\Delta, \Delta_T)]), \qquad (40)$$

which are equivalent to equations (19) and (20) since $\hat{\varepsilon} > \varepsilon_C$ holds in this region.

Now suppose that $r = r_R$ holds. In this case, the derivative in (37) shows that the objective function is strictly increasing in Δ when $\varepsilon_K < \varepsilon_C$ holds and, hence, the solution must satisfy $\varepsilon_K \ge \varepsilon_C$. The optimal amount of term borrowing in this case is characterized by setting (36) to zero, which can be written as

$$r_T = \underbrace{r_R}_{=r} + \frac{(r_X - r_R)}{1 - \theta_X} \{ 1 - G[\varepsilon_C(\Delta_T)] \}.$$

$$\tag{41}$$

Since $\hat{\varepsilon} \leq \varepsilon_C$ holds in this region, this equation is equivalent to (20). Moreover, since the derivative (35) is equal to zero for any value of Δ , we have a solution to the problem as long as $\varepsilon_K \geq \varepsilon_C$ holds, which straightforward calculations show is equivalent to condition (21). Combining the solutions for these two cases yields conditions (19), (20), and (21), as desired.

		Re	ро		
Central Bank	Frame work	Term	Collateral	Lending Facility Collateral	Reserve Requirement
Reserve Bank of Australia	Corridor	1d - 6m	non- HQLA	non- HQLA	no
Banco Central do Brazil	Corridor	1d-30d, 5m, 7m	HQLA	HQLA	no
Bank of Canada	Corridor	1d	HQLA	$\operatorname{non-}$ HQLA	no
ECB	Corridor	1w, 3m, [6m], [3y]	non- HQLA	non- HQLA	yes
Hong Kong SAR	\mathbf{FX}	na	na	HQLA	no
Reserve Bank of India	Corridor	1d	HQLA	non- HQLA	yes
Bank of Japan	Corridor	1d, 4m $[1y]$	non- HQLA	non- HQLA	yes
Bank of Korea	Corridor	1w	HQLA	HQLA	yes
Banco de Mexico	Corridor	1d to 25d	HQLA	HQLA	no
Singapore	\mathbf{FX}	na	$\mathbf{n}\mathbf{a}$	HQLA	no
Sveriges Riksbank	Corridor	1 w	HQLA	non- HQLA	no
Swiss National Bank	Corridor	1 w	HQLA	HQLA	Yes
Bank of England	Corridor	1w, 3m, 3-12m	HQLA	HQLA	voluntary
Federal Reserve	Corridor	1d-14d, <65d	HQLA	$\operatorname{non-}$ HQLA	yes

B Central bank operational frameworks

Source: Monetary policy frameworks and central bank market operations, Markets Committee, June 2008

Table 2: 0	Central	bank	market	operations
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